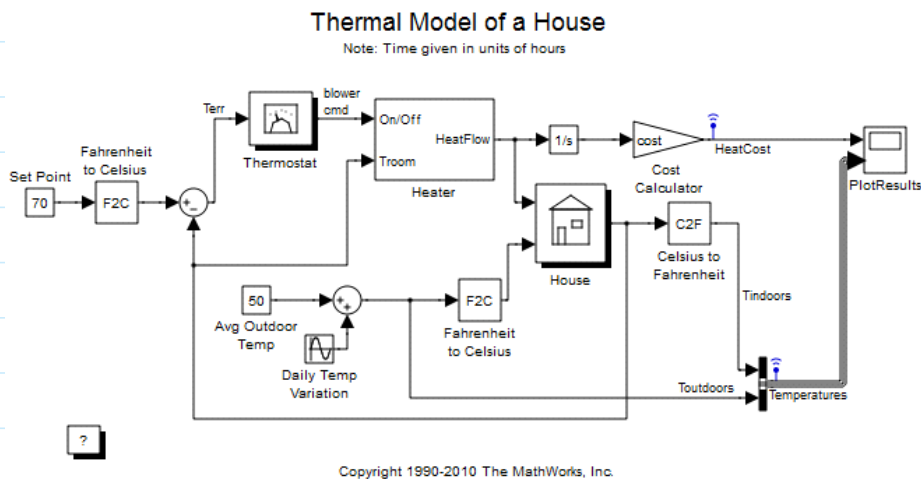


Role of controller : Reduce the error  $e$  (deviation of measured output  $y_m$  from its setpoint  $y_{sp}$ ) to zero (0)

Control modes : define relationship between the controller output  $p$  and the error  $e$

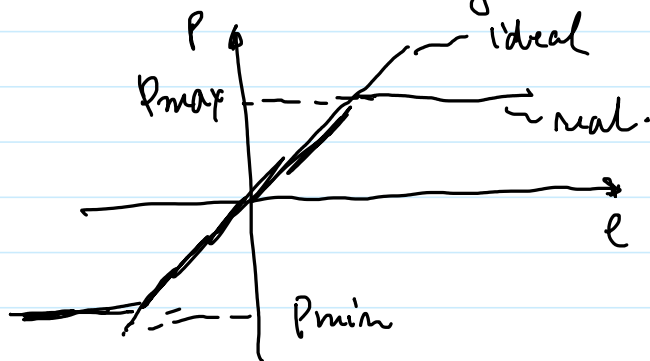
- 1) on-off control (discussed last lecture)



- 2) Proportional control:  $\rightarrow$  output of the controller is proportional to the error.

$$p(t) = \bar{p} + (K_c) e(t) \quad e(t) = y_{sp} - y_m(t)$$

$\bar{p}$  = "bias" = the steady state value of the controller output.



Due to actuator saturation (inherent limitations in actuator capacity - e.g. a valve can only open to 100% close to 0%), the response is not continuous in real systems, it will "plateau" @ min, max values dictated by the actuator capacity

Sign of the gain  $K_c$ ? dictated by the gain of the system to be controlled:

A controller is called

- Direct acting: if it is desired that its output increase ~~th~~ when  $y_m$  increases

output  $\uparrow$  if  $y_m \uparrow$   $k_c < 0$

- Reverse acting

output  $\uparrow$  when  $y_m \downarrow$   $k_c > 0$

The output  $p$  of a proportional controller only depends on the current value of the measured output  $y_m$ . This results in steady-state offset:

the value of  $y$  will never be equal to  $y_m$ , or, equivalently, the control error  $e$  will never be zero.

### ③ Proportional - Integral control:

- Implemented to eliminate steady-state offset.

$$P(t) = \bar{p} + k_c \left( e(t) + \frac{1}{T_I} \int_0^t e(t^*) dt^* \right)$$

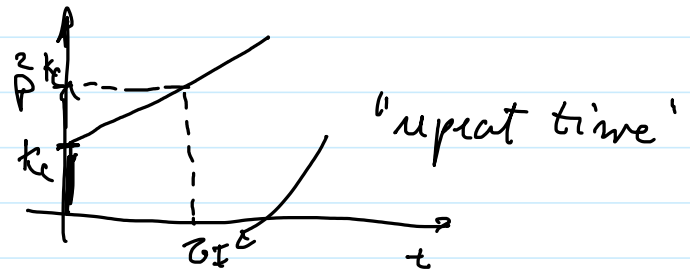
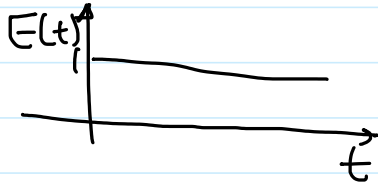
→ operating principle: the controller output will keep changing as long as the error  $e(t)$  has not reached zero.

Transfer fun:

$$\frac{\tilde{P}(s)}{E(s)} = k_c \left( \frac{T_I s + 1}{T_I s} \right)$$

Response of PI controller to step change in error?

$$E(s) = \frac{1}{s}$$



If actuator /controller saturation occurs, the error integral computed by the PI controller will continue to grow. Likewise, the output of the controller,  $p$ , will continue to change. However, b/c actuator saturation, these changes will not affect the operation of the process.

The continued increase in the error integral, which occurs in the presence of saturation is referred to as "integral windup" or "reset windup"

Solution: implement an "anti-windup" mechanism: take the integral component of the controller off-line (i.e., stop integrating the error) when saturation occurs.

Typical situations when windup can occur:

- startup (e.g., empty tank under PI level control)
- large, sudden disturbances (e.g., changes in operating point/ changes in product grade)

② Derivative control:

$$p(t) = \bar{P} + T_D \frac{de(t)}{dt}$$

$T_D$  = derivative time constant  
units = time.

concept: Anticipate (vs. using memory effect)

that is provided by (action) the future behavior of the error signal by considering its rate of change

P control : acts on current deviation of  $y_m$  from  $y_{sp}$

I : based on past history,

(D) - control: valuable when sudden changes in a controlled variable occur.

Q: Does D-control by itself lead to zero offset? No : the output of a derivative controller does not change unless the error  $e(t)$  changes in time.

→ D-control is never used by itself, only in combination with P, PI control.

→ Potential issue: if data are noisy, derivative of the error is difficult to calculate

→ need data smoothing

→ Transfer function of the ideal PD controller.

$$\tilde{p}(t) = k_c \left( e(t) + T_D \frac{de}{dt} \right)$$

$$\frac{\tilde{P}(s)}{E(s)} = k_c (1 + T_D s) \leftarrow \text{Not physically realizable.}$$

Transformed version:

$$\frac{\tilde{P}(s)}{E(s)} = k_c \left( 1 + \frac{T_D s}{\alpha T_D s + 1} \right)$$

derivative filter form

$$\alpha = 0.05 \dots 0.2$$

③ Proportional - Integral - Derivative (PID) Control

= Vast proportion of industrial controllers

- Combination of P, I, D control modes. ~~PI~~

- Can take several functional forms:

$$p(t) = \bar{p} + k_c \left[ e(t) + \frac{1}{T_I} \int_0^t e(t^*) dt^* + T_D \frac{de(t)}{dt} \right]$$

T.F:

$$\frac{\tilde{P}(s)}{E(s)} = k_c \left[ 1 + \frac{1}{T_I s} + T_D s \right]$$

↑ parallel form

Series form:

— w/ derivative

Series form:

$$\frac{\tilde{P}(s)}{E(s)} = K_c \left( \frac{z_I s + 1}{z_I s} \right) \left( \frac{z_D s + 1}{2 z_D s + 1} \right)$$

↑ implemented in the analog controller.

↙ w/ derivative filter,